AMERICAN UINVERSITY OF BEIRUT
FACULTY OF ENGINEERING AND ARCHITECTURE
EECE 460 Control Systems
Fall 2004-2005
Quiz II SOLUTION
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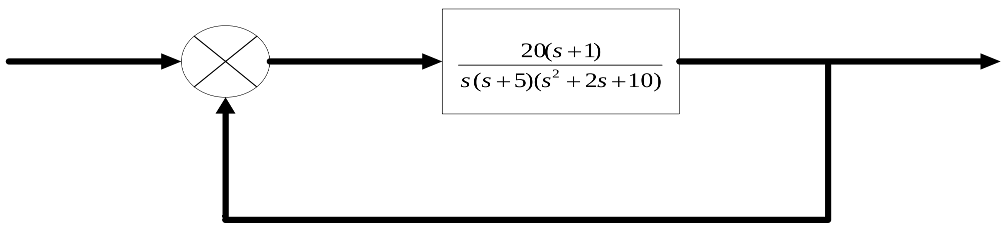
1.5 Hours, January 7, 2005
Total of 100 points; Open Book Exam, 3 pages
YOU MUST RETURN THIS EXAM WITH YOUR ANSWER BOOKLET

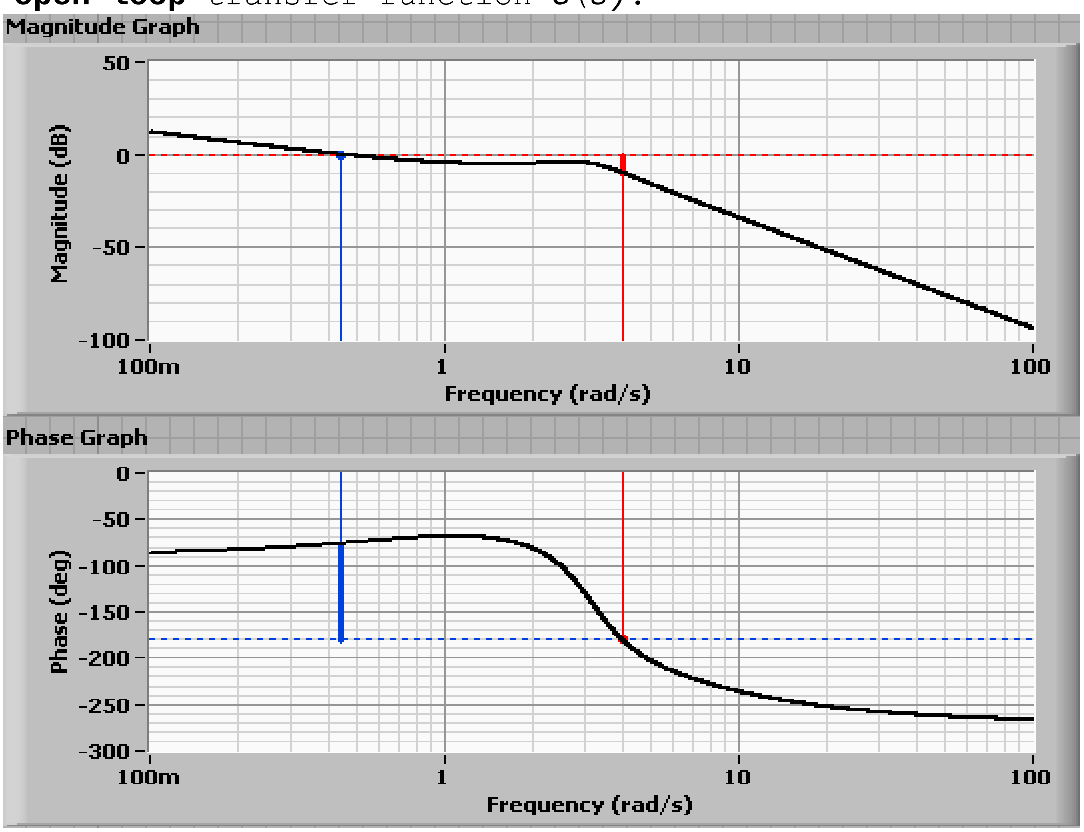
Problem 1 (30 points):

Consider the system shown in the Figure below.



Control system with unity feedback

a) Approximate on a semi-log paper the Bode diagram of the **open-loop** transfer function $G(s)$.



b) Based on part (a), approximate graphically the phase margin and gain margin

The phase margin, gain margin, phase crossover frequency, and gain crossover frequency values are

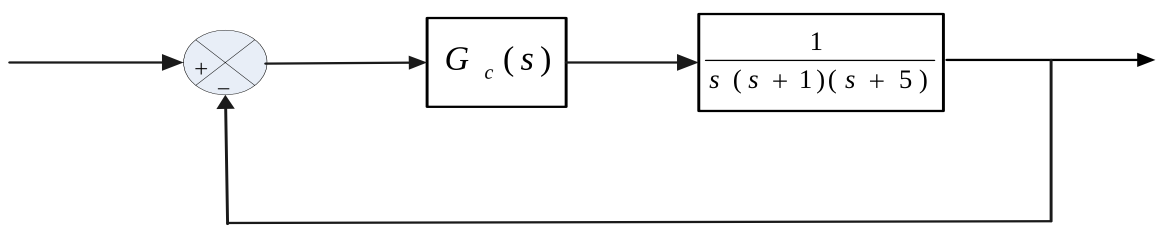
G.M. Frequency	Gain Margin
4.0148	9.9404
P.M. Frequency	**Phase Margin**
0.4426	103.6573

c) Using the results of part (b), is the **Closed-loop** transfer function stable? Justify.

System is stable since the open loop transfer function is Minimum Phase and the P.M. and G.M. are positive.

Problem 2 (30 points):

Consider the system shown in the Figure below.

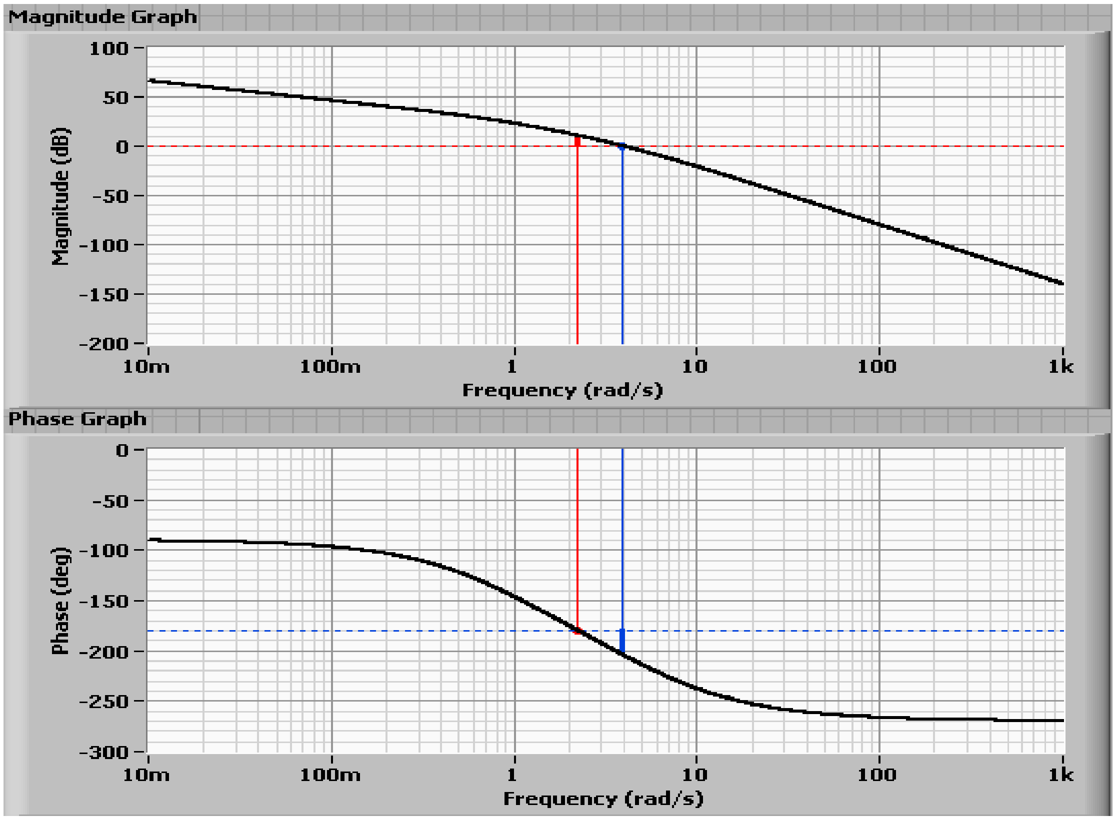


Control System with series controller

Let us assume that the compensator $G_c(s)$ is a lag-lead and has the following form:

$$G_c(s) = K_c \frac{(s+\frac{1}{T_1})(s+\frac{1}{T_2})}{(s+\frac{\beta}{T_1})(s+\frac{1}{\beta T_2})}$$

The uncontrolled open loop system bode plots with K_c = 100 corresponding to the static velocity error constant K_v is 20 sec^{-1} are



a) Design the compensator (if needed) such that phase margin is at least 60°, and gain margin is not less than 8 dB.

From this Bode diagram we find the phase crossover frequency to be ω = 2.25 rad/sec. Let us choose the gain crossover frequency of the designed system to be ω = 2.25 rad/sec so that the phase lead angle required at ω = 2.25 rad/sec is 60°.

Once we choose the gain crossover frequency to be 2.25 rad/sec, we can determine the corner frequencies of the phase lag portion of the lag-lead compensator. Let us choose the corner frequency $1/T_2$ to be one decade below the new gain crossover frequency, or $1/T_2 = 0.255$. For the lead portion of the compensator, we first determine the value of β that provides $\varphi_m = 65°$, (5° added to 60°.) Since

$$\sin\phi_m = \frac{1-\frac{1}{\beta}}{1+\frac{1}{\beta}} = \frac{\beta-1}{\beta+1}$$

We find β = 20 corresponds to 64.7912°. Since we need 65° phase margin, we may choose β = 20. Thus

$$\beta = 20$$

Then, the corner frequency $1/(\beta T_2)$ of the phase lag portion becomes as follows:

$$\frac{1}{\beta T_2}=\frac{1}{20\,x\,\dfrac{1}{0.225}}=\frac{0.225}{20}=0.01125$$

Hence, the phase lag portion of the compensator becomes as

$$\frac{s+0.225}{s+0.01125}=20\frac{4.4444s+1}{88.8889s+1}$$

For the phase lead portion, we first note that

$G_1(j\ 2.25)$ = 10.35 dB.

If the lag-lead compensator contributes -10.35 dB at ω = 2.25 rad/sec, then the new gain crossover frequency will be as desired. The intersections of the line with slope +20 dB/dec [passing through the point (2.25,-10.35 dB)] and the 0 dB line and -26.0206 dB line determine the corner frequencies. Such intersections are found as
ω = 0.3704 and ω = 7.4077 rad/sec, respectively.

Thus, the phase lead portion becomes

$$\frac{s+0.3704}{s+7.4077}=\frac{1}{20}\left(\frac{2.6998s+1}{0.1350s+1}\right)$$

Hence the compensator can be written as

$$G_c(s)=100\left(\frac{4.4444s+1}{88.8889s+1}\right)\left(\frac{2.6998s+1}{0.1350s+1}\right)$$

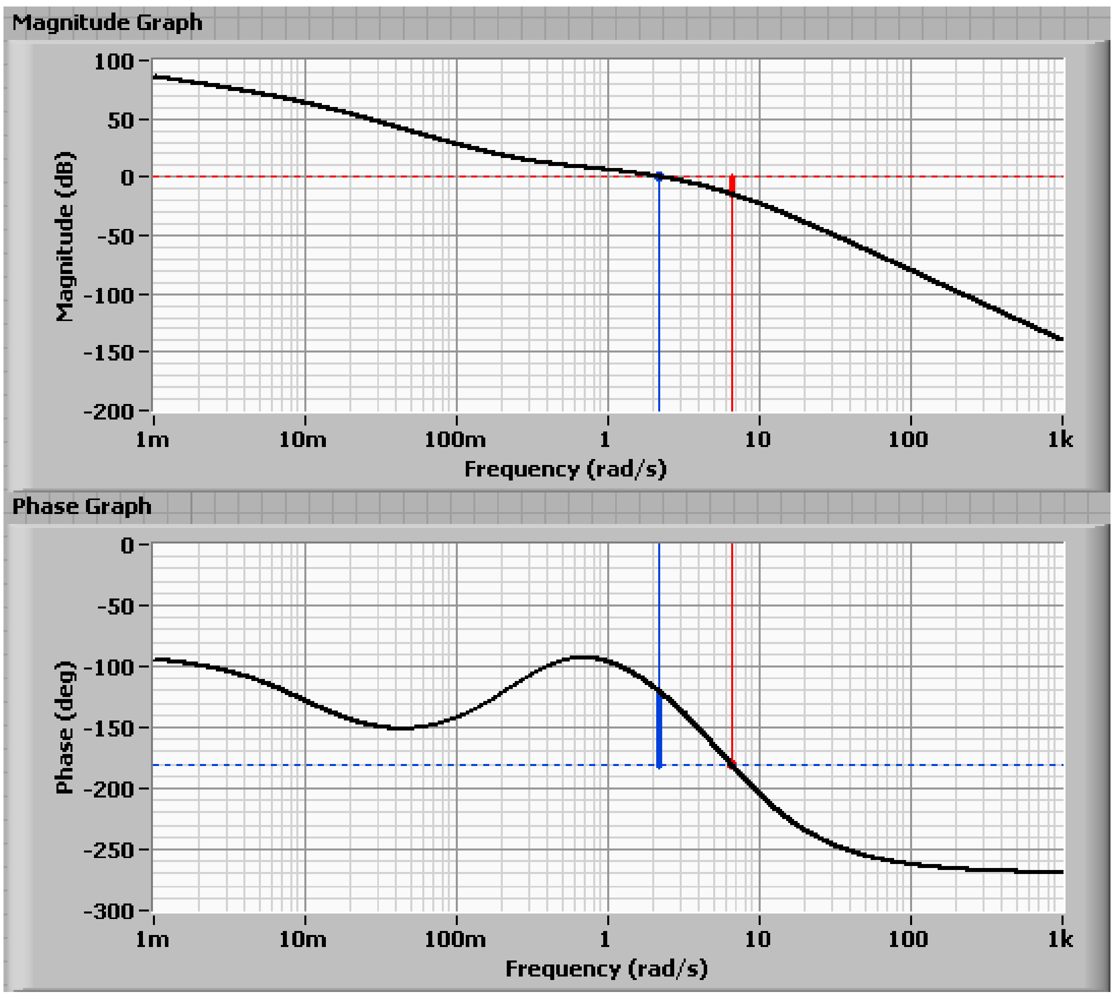
$$=100\left(\frac{s+0.225}{s+0.01125}\right)\left(\frac{s+0.3704}{s+7.4.077}\right)$$

Then the open-loop transfer function $G_c(s)G(s)$ becomes as follows:

$$G_c(s) = 100\left(\frac{4.4444s+1}{88.8889s+1}\right)\left(\frac{2.6998s+1}{0.1350s+1}\right)\frac{1}{s(s+1)(s+5)}$$

$$= \frac{1199.90s^2 + 714.42s + 100}{12s^5 + 161.0239s^4 + 595.1434s^3 + 451.1195s^2 + 5s}$$

b) Verify the effectiveness of the controller in meeting the desired specs.



From this diagram we find that the phase margin is approximately 60° and gain margin is 14.35 dB. The static velocity error constant is 20 sec^{-1}.

Problem 3(40 points):

Consider the servo system described by a state model LTI. Matrices A, B, and C are given as:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -5 & -6 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad C = [1 \ 0 \ \ 0]$$

The nonzero output reference is r(t).

a) Is the system fully controllable? Verify.

Q = [B AB A2B] has full rank, system is controllable.

b) Determine **if possible** the feedback gain constant vector of a Pole Placement Controller using state feedback [k1, k2, k3] such that the desired closed loop poles are:

S1 = -2 + j4 S2 = -2 – j4 S3 = -10
we assume that all 3 states are available for feedback

K = [200 55 8]

c) Is the system fully observable? Verify.

O = [C CA CA2]T full rank, system is observable

d) Realistically only the output y(t) is available for feedback. Design an observer (**if possible**) to estimate on-line the internal states using only system output y(t) and input u(t).

Since desired poles of Observer are not supplied, we need to choose them (non-uniquely) much faster (5x) than the system in order for the observer estimation to converge faster than controlled process in need of good estimation. Hence, for every choice there corresponds a gain vector.